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Self-oscillating regimes of nucleate, transition and film boiling

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Abstract—The models of unstable nucleate and film boiling are suggested. The model of bulk boiling takes into account the processes of heat transfer from an external source, nucleation, bubble growth and withdrawal of bubbles from the metastable zone. The surface of neutral stability for stationary boiling regimes is obtained and bifurcation analysis of the threshold of instability is carried out. The amplitude and frequency of non-linear self-oscillating regimes are calculated. The analysis of unstable boiling on a heating surface is based on the model of conjugate heat transfer problems in a plate of finite thickness and in a boundary layer. The conditions of transfer from pulsating nucleate boiling to the dry patch formed during boiling with repeated rewetting are determined. Theoretical results are compared with available experimental data. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

The behaviour of boiling liquids is usually complicated by a number of instabilities of various physical origins resulting in self-oscillations, pulsations and crisis phenomena [1, 2]. A great interest to the analysis of these problems is dictated by both important practical applications of boiling systems and purely scientific goals to study a poorly known field of heat transfer under the conditions of high metastability and complex phase interaction.

Hydrodynamic and thermohydrodynamic instability of two-phase metastable systems can have a local character. These are: oscillations of a single bubble in the field of variable pressure or acoustic field [3, 4], the well-known Helmholtz and Taylor instabilities [5, 6], collapse of bubbles [7, 8], etc. As distinct from the local instability, macro-instability affects the whole system: these are density-wave oscillations [9], acoustic oscillations [10], kinematic waves [11] or oscillations of pressure overfall [12]. Oscillations in two-phase systems can be smooth, nearly harmonic (wave flows of liquid in film boiling in a gravitational field on a heated surface being turned down), relaxational and ruptured (processes of nucleate, transition and film boiling), or can have complex stochastic character (turbulent flows in tubes). Since the behaviour of a two-phase system is very frequently determined by a contribution of several elementary physical mechanisms of instability, the latter should be treated as a complex phenomenon when all the mechanisms operate simultaneously. The analysis of the dynamic behaviour of two-phase systems is generally based on the fundamental differential conservation equations written out for each of the phases and supplemented with conjugate conditions on the interphase surfaces. However, it is impossible to obtain a full solution of these equations even in a stationary statement. Attempts to simultaneously take into account different instability mechanisms result in poorly traced and highly cumbersome numerical calculations [13]. The latter, as a rule, give no chance either to single out the influence of different instability mechanisms or to reveal the main reason of instability in boiling systems. Thus, it seems natural to consider such factors separately in the limits of the appropriate model statements.

A special place is occupied by the physical mechanism of instability in boiling liquids caused by a mutual interdependence between the external heating of the system and its cooling due to the absorption of the latent heat of vaporization during the evolution of a boiling particulate system. Depending on the method of external heating and the specifics of the system, this mechanism manifests itself both under the conditions of nucleate or film boiling on the heated surface and in the process of bulk boiling inside a volume of superheated liquid.

Under the conditions of the bulk superheating of liquid, when nucleation on solid surfaces is absent, the majority of nucleation centres occur in the way of fluctuation both on deep intrusion into the metastable region and at moderate metastability, if there are "boiling kernels", i.e. the zones of local superheating whose linear size is much in excess of the mean bubble radius [14]. At the initial stages of the process the activation at small superheating of the heterogeneous centres of nucleation that exist in liquid can be essential. However, the role of such centres relaxes in due time because of their withdrawal with bubbles.

Though in the process of bulk boiling, oscillations and pulsations are not so widely spread as in heterogeneous boiling on heated surfaces, their practical importance is not to be underestimated. Just these phenomena may be responsible for periodic throws

NOMENCLATURE

a_1, a_2	thermal diffusivity in plane and liquid
$B(u)$	function introduced in equation (14)
B'	coefficient introduced in equation (4)
c	thermal capacity of two-phase system, equation (2)
c_1	thermal capacity in plate, equation (24)
$f(t,r)$	bubble size distribution function
g	acceleration due to gravity
N	number concentration of molecules in liquid
N_A	Avogadro number
M	molecular weight
L	latent heat for vaporization
P_0	equilibrium pressure
p'	pressure in liquid
Δp	pressure overdrop
r	bubble radius
s	dimensionless bubble radius
t	time
t_*	dimensionless time
T_0	saturation temperature
T_1, T_2, T_3	temperature in plate, liquid and vapour film, respectively
T_w	wall temperature.

Greek symbols

α	effective heat transfer coefficient
γ	bubble withdrawal rate
λ_0	thermal conductivity in plate
λ'	thermal conductivity in liquid
μ', μ''	dynamic viscosity in liquid and vapour, respectively
ν', ν''	kinematic viscosity in liquid and vapour, respectively
ρ, ρ_0, ρ', ρ''	density of two-phase system, plate, liquid and vapour, respectively
σ	surface tension
Ψ	coefficient introduced in equation (4)
Ω	functional

$$\int_0^{\infty} \exp\left(-\int_0^s (z)\,\mathrm{d}z\right)\frac{r^2(s)\,\mathrm{d}s}{A(s)}.$$

Superscript

0	corresponds to neutral stability curve.

Subscript

s	corresponds to steady-state regime.

of a two-phase mixture in modern nuclear reactors, caused by sudden boiling up of large masses of liquid. Such periodic throws of a two-phase mixture and 'geysers' are able to initiate some other instabilities, e.g. to cause the density and pressure waves and shell flows in steam generators of fast reactors. Thus, the instability of phase transition, having in essence the thermal nature, can cause thermohydrodynamic instabilities.

Self-oscillating processes of bulk boiling are widely spread not only in modern technology, but also in nature. For example, periodic vapour throws in geysers, hot springs and fumaroles, being important for geothermal energetics, are common knowledge. The modern model of a geyser is based just on the idea of a sudden boiling up of water in underground reservoirs [15].

A principal model describing the instability of stationary regimes of bulk boiling and formation of self-oscillating regimes was suggested for the first time in refs. [16, 17]. These papers present a criterion of neutral stability for a stationary boiling regime and characteristics of self-oscillations originating in the regions of instability, which are studied in the approximation of ruptured relaxational oscillations for the case of a very strong dependence of nucleation rate on superheating. The present paper develops the ideas of refs. [16, 17] and eliminates the shortcomings of these papers relating to physical and mathematical modelling of the process.

As regards heterogeneous boiling on heated surfaces, pulsations and oscillations were observed repeatedly. The reasons for the origination of these phenomena are diverse. The problems of the stability of boiling regimes and oscillating phenomena may be treated locally, by considering the processes on a single nucleation site, or generally, by studying the instability on a considerable part of a heated surface. The problems of boiling stability on a separate nucleation centre and corresponding local temperature pulsations were studied by Mesler *et al.*, Nesis *et al.* and Tolubinsky *et al.* [18–20]. The papers [19, 20] showed that unstable functioning of a nucleation centre is caused by a temporary damping of its activity due to a full vapour condensation in a cavity and its flooding by liquid and by random convective flows in liquid in the waiting period. Fluctuations of the surface temperature and unstable heat transfer regimes enveloping the whole boiling surface are much more important. Such processes are observed most frequently in boiling liquid metals that easily wet the boiling surface (this causes large waiting times and high superheatings). Pulsations of the surface temperature caused by turbulent flows and heat transfer in a viscous underlayer were investigated in refs. [21, 22]. Oscillations and pulsations of the temperature of heat transfer surfaces in the process of boiling can result from the peculiarities of hydrodynamic processes occurring in power equipment and are most frequently encountered in annular dispersed two-phase flows;

such processes were studied in refs. [23, 24]. Characteristics of temperature oscillations resulting from the oscillations of the liquid film surface in the process of boiling on a vertical surface are obtained in ref. [25]. Temperature oscillations caused by the crises of heat transfer are of special importance. The paper [26] presents analytical models for calculating temperature oscillations in heat transfer tubes caused by a critical heat flux. The oscillating nature of convective conditions on the liquid–vapour interface is modelled under the assumption that a number of vortex-type wetting zones exist on the tube circumference. Surfaces poorly provided with nucleation sites are conducive to the realization of the third boiling crisis, when the regime of a mono-phase convection periodically changes into spontaneous boiling of metastable liquid.

The present paper shows that self-oscillations in the process of heterogeneous boiling on a heated surface can arise even under the conditions of stable operation of nucleation sites. Such regimes are caused by a very strong dependence of the density of nucleation sites on the temperature force. The principal physical mechanism of self-oscillations is the same as that in bulk boiling, i.e. it consists of non-linear interaction between heat release and heat absorption during phase transition. It is very important that the limits of the model suggested give an opportunity to describe the process of unstable transition boiling when the heating surface is periodically wetted by liquid and vapour.

INSTABILITY AND SELF-OSCILLATIONS IN THE PROCESS OF BULK BOILING

The model

Consider a sufficiently large volume of a superheated liquid. Suppose that the nucleation centres have the activation nature, which is characterized by a strongly non-linear dependence of nucleation rate on superheating. To simplify the mathematical description, the parameters of a two-phase system are considered to be homogeneous throughout the whole volume, and the thermophysical characteristics of the both phases as constant. This assumption, however, is not a principal one in the sense that all the calculations given below can be rather easily adapted to situations when thermophysical characteristics are certain functions of temperature. The homogeneous heating of liquid can be carried out by means of electric current or radiation. In a more general case the assumption on the homogeneity of parameters is correct for non-uniform heating, if conductive or convective (organized by means of the mixing of liquid due to its turbulization by moving bubbles) heat transfer in the volume is sufficiently intensive. The time of the nucleation rate relaxation to its stationary value in a metastable liquid

$$\frac{8\pi N_A \rho'' kT}{M p_0 (p_0 - p')} \sim 10^{-9}\,\mathrm{s} \tag{1}$$

is much smaller than the characteristic time scale of the process. Thus it is possible in what follows to use the results of the stationary nucleation theory.

Under these assumptions the heat balance equation for the unit volume of the two-phase system and the kinetic equation describing the evolution of the bubble size distribution function assume the following form:

$$\rho c \frac{\mathrm{d}T}{\mathrm{d}t} = \alpha(T_m - T) - \rho' L \int_{r_*}^{\infty} \frac{\mathrm{d}}{\mathrm{d}t}\left(\tfrac{4}{3}\pi r^3\right) f(t, r)\,\mathrm{d}r \tag{2}$$

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial r}\left(\frac{\mathrm{d}r}{\mathrm{d}t} f\right) + \gamma(r) f = 0 \quad b\frac{\mathrm{d}r}{\mathrm{d}t} f\bigg|_{r=r_*} = J(T - T_0/T_0). \tag{3}$$

We neglect random fluctuations of the bubble growth rate $\mathrm{d}r/\mathrm{d}t$ in equation (3). They are usually taken into account by including appropriate diffusion terms into the boundary condition and equation (3).

The nucleation rate J is determined by the Dering–Volmer and Frenkel–Zeldovich–Kagan theories:

$$J = NB' \exp(-W/kT)$$

$$W = \frac{16\pi\sigma^3 \Psi}{3(p_0 - p')^2 (1 - \rho''/\rho')^2}. \tag{4}$$

The Dering–Volmer theory assumes that the growth rate of a vapour cluster is controlled by evaporation and condensation rates, and the influence of viscous and inertia forces is negligible. The Frenkel–Zeldovich–Kagan theory describes the nucleation rate more correctly, taking account of viscous and inertia forces as well as the evaporation and heat supply rates. The difference between these theories consists in the form of the multiplier B', which is a weak function of the state of the system. If heterogeneous centres of nucleation having the activation nature are present in the volume, it is necessary to choose an appropriate coefficient Ψ in equation (4) in order to coordinate the theoretical values of superheating with experimental ones. Note that reliable data on the values of the coefficient Ψ is accessible only for a few types of heterogeneous nucleation centres [27].

At a moderate pressure and superheating one can represent the difference $p_0 - p'$ as a value proportional to $T - T_0$. The use of the Clapeyron–Clausius equation yields

$$p_0 - p' = \frac{L\rho'\rho''}{\rho' - \rho''}\,\frac{T - T_0}{T_0}. \tag{5}$$

Evidently, the rate of the withdrawal of bubbles out of the volume considered depends essentially on their size. If the withdrawal is accomplished by the buoyancy forces in the field of gravitation, in the approximation of free growth and moving of bubbles (when one can neglect thermohydrodynamic interaction of bubbles and the influence of the walls), it is reasonable to assume

$$\gamma(r) = \varkappa V(r), \tag{6}$$

where V is the rate of the approach of a single bubble to the surface, and $\varkappa$ is the coefficient of proportionality determined by the size of the metastable region. For very small bubbles, corresponding to the Reynolds numbers

$$Re = \frac{Vr}{\nu'} < 1 \tag{7}$$

and having a spherical form, there is the Hadamard–Rybchinsky solution which takes account of the mobility of the bubble-liquid interface

$$V(r) = \frac{2}{3}\frac{gr^2}{\nu'}(1-\rho''/\rho')\frac{\mu'+\mu''}{2\mu'+3\mu''}. \tag{8}$$

When $\mu'/\mu'' \to 0$, which corresponds to immovable (hardened) interface, equation (8) gives the Stokes formula

$$V(r) = \frac{2}{9}\frac{gr^2}{\nu'}(1-\rho''/\rho'). \tag{9}$$

When $Re > 1$, for the further analysis use will be made of the formulae from ref. [28] obtained by using the similarity theory

$$V(r) = \begin{cases} 100r\left(\dfrac{\sigma}{\mu'\nu}\right)^{2/3}\left[\dfrac{g^2r(\rho'-\rho'')}{\sigma}\right]^{1/2}, & r < r_0 \\ \left[\dfrac{\sigma}{r(\rho'+\rho'')}+\dfrac{gr(\rho'-\rho'')}{\rho'+\rho''}\right]^{1/2}, & r \geqslant r_0 \end{cases}$$

$$r_0 = 0.1\left(\frac{\mu'\nu}{\sigma}\right)^{1/3}\left[\frac{\sigma}{g(\rho'-\rho'')}\right]^{1/2}. \tag{10}$$

Equations (10) were verified by Malenkov [28] for a number of liquids; he obtained a good correspondence with experimental data in the interval $Re \sim 1$–450. Though equations (10) correspond to the bubbles being in equilibrium with liquid, they can also be recommended for nonequilibrium bubbles, excluding the smallest ones, whose contribution to the integral term in equation (2) is negligible.

There is a great variety of theoretical and empirical formulae describing the bubble growth rate. When the bubble growth is limited by the inertia of liquid, one obtains

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \left(\frac{2}{3}\frac{\Delta p}{\rho'}\right)^{1/2}. \tag{11}$$

For sufficiently large bubbles, whose growth is controlled by the heat supply rate for evaporation, the following approximate correlation is valid in a wide interval of the Jacobs numbers Ja [29]

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{6a'}{\pi r}Ja^2\left[1-\frac{1}{2}\left(\frac{\pi}{6Ja}\right)^{2/3}+\frac{\pi}{6Ja}\right], \quad Ja = \frac{c'\rho'\Delta T}{L\rho''}. \tag{12}$$

The evolution laws listed are asymptotic ones: at the initial stage the bubble growth is limited by inertia forces, and later, by heat transfer. Usually all the practical situations are located between equations (11) and (12). However, under some specific conditions the bubble growth is fully controlled by one of equations (11) or (12). For example, under high superheating, low pressure and high thermal conductivity, which is typical for liquid metals, formula (11) is valid during almost the whole period of growth. Under the conditions of low superheating, high pressure and low thermal conductivity the dynamics of the bubble growth is close to formula (12).

Analysis and results

Let us introduce new variables

$$u = \frac{T-T_0}{T_0} \qquad s = \gamma_0\int_0^r \frac{A(r)\,\mathrm{d}r}{B_s}$$

$$t_* = \gamma_0\int_0^t \frac{B(u(z))}{B_s}\mathrm{d}z \qquad \Gamma = \frac{\gamma}{\gamma_0} \tag{13}$$

where γ_0 is some typical value of the function γ; if $(\sigma g/\rho')^{1/4}$ is the scale of V, i.e. $V = (\sigma g/\rho')^{1/4}U$, then $\gamma_0 = \varkappa(\sigma g/\rho')^{1/4}$. In equations (13) the following general form for bubble growth rate is assumed

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{B(u)}{A(r)}. \tag{14}$$

By using the method suggested in ref. [30] the bubble size distribution density in the variables of equations (13) can be obtained

$$f(t_*, s) = \frac{J(u(t_*-s))}{B(u(t_*-s))}\exp\left(-\int_0^s \frac{B_s\Gamma(s-z)\,\mathrm{d}z}{B(u(t_*-z))}\right)A(s). \tag{15}$$

Equation (15) was derived in the assumption that the boiling process was considered on the developed asymptotic stage, when the influence of initial conditions and initial bubble concentration in the volume was inessential (initial conditions are substituted in this case by the conditions of periodicity), and the crytical nuclei radius was negligibly small, i.e.

$$s_* = \gamma_0\int_0^{r_*}\frac{A(r)\,\mathrm{d}r}{B_s} \approx 0. \tag{16}$$

By using equation (15) in equation (2) it is possible to obtain, after a simple transformation, an integro-differential functional equation governing the evolution of relative superheating in the system

$$\rho'c'\gamma_0\frac{B(u)\,\mathrm{d}u}{B_s\,\mathrm{d}t_*}+\alpha(u-u_\mathrm{m}) + \frac{4\pi\rho''LB_sB(u)}{\gamma_0T_0}\times\int_0^\infty \frac{J(u(t_*-s))}{B(u(t_*-s))} \times\exp\left(-\int_0^s\frac{B_s\Gamma(s-z)\,\mathrm{d}z}{B(u(t_*-z))}\right)\frac{r^2(s)\,\mathrm{d}s}{A(s)} = 0. \tag{17}$$

It is convenient to introduce the new variable and the Stanton parameters

$$x = (u-u_s)/u_s$$

$$St = \alpha(u_m-u_s)/\rho'c'\gamma_0 u_s$$

$$St_m = \alpha u_m/\rho'c'\gamma_0 u_s. \qquad (18)$$

The effective Stanton numbers St and St_m characterize the interaction between the integral heat emission and the heat release during the growth of a bubble.

Taking account of equations (18), equation (17) assumes the following form

$$\frac{B(u)\,\mathrm{d}x}{B_s\,\mathrm{d}t_*}+(St_m-St)x-St+\frac{StB(u)}{\Omega}$$

$$\times\int_0^\infty \frac{J(u(t_*-s))}{J_sB(u(t_*-s))}$$

$$\times\exp\left(-\int_0^s \frac{B_s\Gamma(s-z)\,\mathrm{d}z}{B(u(t_*-z))}\right)\frac{r^2(s)\,\mathrm{d}s}{A(s)}=0. \qquad (19)$$

The bubble size distribution function and the superheating corresponding to the stationary boiling regime are described by the stationary analogues of equations (15) and (19)

$$f(s)=\frac{A(s)J_s}{B_s}\exp\left(-\int_0^s\Gamma(z)\,\mathrm{d}z\right),$$

$$\alpha(u_m-u_s)=\frac{4\pi\rho'LB_sJ_s\Omega}{\gamma_0T_0} \qquad (20)$$

(note that transcendental equations (20) can serve for determining the value of α, if the stationary superheating u_s is known).

The stationary boiling regime (20) may turn out to be unstable with respect to random fluctuations of temperature. The problem of stability is of great importance, since the stability break may cause changes in both the course of the process and its results. Analysis of stability with respect to small fluctuations of temperature can be carried out on the basis of linearized equation (19):

$$\frac{\mathrm{d}x}{\mathrm{d}t_*}+x(St_m-St+RSt)+\frac{G'-RSt}{\Omega}$$

$$\times\int_0^\infty x(t_*-s)\exp\left(-\int_0^s\Gamma(z)\,\mathrm{d}z\right)\frac{r^2(s)\,\mathrm{d}s}{A(s)}$$

$$+\frac{RSt}{\Omega}\int_0^\infty\left(\int_0^s\Gamma(s-z)x(t_*-z)\,\mathrm{d}z\right)$$

$$\times\exp\left(-\int_0^s\Gamma(z)\,\mathrm{d}z\right)\frac{r^2(s)\,\mathrm{d}s}{A(s)}=0, \qquad (21)$$

where the following parameters are introduced

$$G'=StG$$

$$G=\left.\frac{u_s\,\mathrm{d}\ln J}{\mathrm{d}u}\right|_{u=u_s}$$

$$R=u_s\left.\frac{\mathrm{d}\ln B}{\mathrm{d}u}\right|_{u=u_s} \qquad (22)$$

where G is the Gibbs number.

Expressing the temperature perturbation x in the form $x = x_0\exp(\gamma t_*)$, the following complex equation for the parameter λ is obtained:

$$\lambda+St_m+St(R-1)+\frac{G'-RSt}{\Omega}$$

$$\times\int_0^\infty\exp(-\lambda s-\int_0^s\Gamma(z)\,\mathrm{d}z)\frac{r^2(s)\,\mathrm{d}s}{A(s)}$$

$$+\frac{RSt}{\Omega}\int_0^\infty\left(\int_0^s\Gamma(s-z)\exp(-\lambda z)\,\mathrm{d}z\right)$$

$$\times\exp\left(-\int_0^s\Gamma(z)\,\mathrm{d}z\right)\frac{r^2(s)\,\mathrm{d}s}{A(s)}=0. \qquad (23)$$

When $\gamma = i\omega$, where ω is the rational quantity, equation (23) represents the neutral stability surface. It was analysed for the both above-indicated bubble growth laws (11) and (12). The traces of the neutral stability surface $G' = G'^0(St_m, St, R)$ in the plane of the parameters G' and St_m for various values of St and R are shown in Fig. 1. Since only the values $u_m \geqslant u_s$ are physically real, the neutral stability curves shown in Fig. 1 should be treated only for $St_m \geqslant St$. The region of instability is determined by the inequality $G' > G'^0$. The instability is caused by the competition between the processes of heat supply, latent heat release during bubble growth, nucleation and bubble withdrawal from the metastable zone. Figure 1 demonstrates that in spite of high metastability, provided that $G' < G'^0$ (the Gibbs number G is quite small in this region), the stationary regimes appear to be stable in the region indicated. Thus, the stability breaks only on the attainment of a sufficiently large slope in the nucleation rate as a function of superheating (the parameter G' is proportional to $\mathrm{d}J/\mathrm{d}u$ at $u = u_s$). When the image point in the parametric space passes over the neutral stability boundary along the line at which St_m is constant (i.e. in the vertical direction), the Gibbs number increases (i.e. the metastability decreases), but this is caused by the growth of the derivative $(\mathrm{d}J/\mathrm{d}u)_{u=u_s}$ and is not connected with the changes in the nucleation rate. Hence the physical reason of the instability is a strongly nonlinear dependence of nucleation rate on superheating.

One can easily see that when the bubble growth is limited by the liquid inertia, the stability region is much narrower than in the bubble growth limited by the heat supply rate for evaporation. This can be explained by the higher bubble growth rate according to the inertia law: the bubbles grow more quickly to

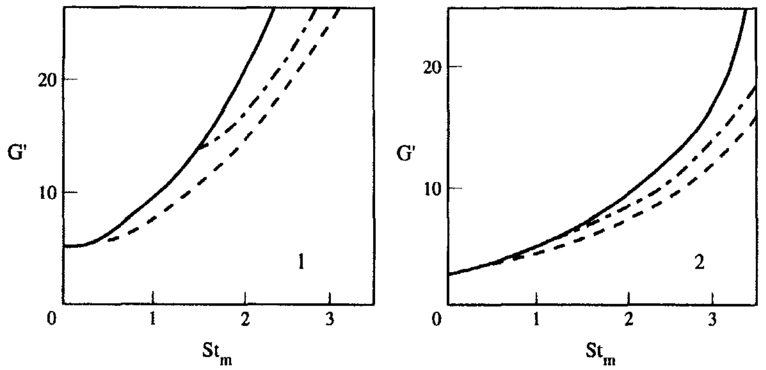

Fig. 1. Neutral stability curves in the plane (G', St_m); solid curve, $St = St_m$; dashed curve, $St = 0.5$; dashed-dotted curve, $St = 1.5$; 1, the bubble growth rate is described by equation (12), 2, by equation (11).

a sufficiently large size and, hence, release the superheating more rapidly, thus destabilizing the system. The instability has an oscillating nature. Figure 2 presents the dependence of the oscillation period on the neutral stability surface on the Stanton number.

The next significant problem concerns the character and properties of new unsteady regimes originating in consequence of the instability of the steady regimes. Since the instability is of an oscillating nature, one has to expect that either an auto-oscillating or a chaotic pulsating regime develops, depending on whether the break of stability of the stationary regime belongs to the 'soft' or 'hard' type. The following reasoning explains the origination of auto-oscillations from the physical point of view. Suppose that the superheating in the volume is slightly increased. This causes an increase of the nucleation rate. Under sufficiently high superheating, the dependence of the nucleation rate on the metastability is very sharp, thus leading to a spontaneous origination of nuclei. Under certain conditions, the growth of bubbles formed may lead to a decrease of the superheating being not compensated by the external heating. Such a decrease brings about a sharp decrease in the nucleation rate and, consequently, some reduction in the inner release of heat. After a period of time, this leads to a decrease of the total number of bubbles in the volume. Then the superheating due to a continuously acting heat source grows again, which in turn gives rise to a new enhancement of the nucleation and bubble growth rates. This reduces the superheating again, and thus an auto-oscillating cycle is established.

To confirm this idea, the bifurcation analysis of integrodifferential equation (19) was undertaken. On the threshold of instability the analysis was carried out by the perturbation methods by using expansions with respect to small supercriticality $(G'-G'^0)/G'^0$ (for the detailed description of this technique see ref. [30]). Far from the instability boundary, equation (19) was analysed numerically by the Eitken–Steffensen iterative method [31]. The results obtained show that the instability develops according to the 'soft' type, and the Landau–Hopf bifurcation of the stationary regime takes place. Thus, the secondary self-oscillating regime forms, the amplitude of which grows while the frequency decreases with an increase of the supercriticality (Fig. 3). However, the oscillation amplitude increases rapidly with an increase of the supercriticality, which makes the region of validity of slightly nonlinear self-oscillations very narrow (as distinct from slightly nonlinear almost harmonic self-oscillations in the processes of continuous crys-

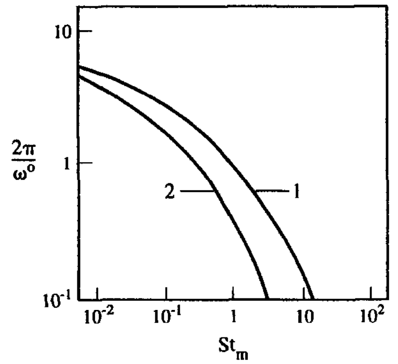

Fig. 2. The oscillation period on the neutral stability curves for $St = St_m$; 1, equation (12), 2, equation (11).

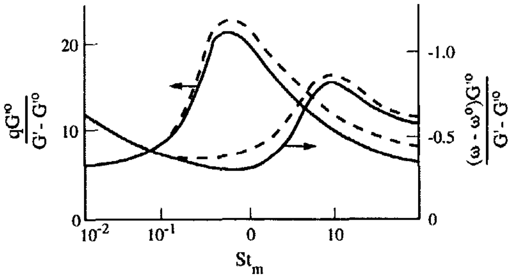

Fig. 3. The squared oscillation amplitude of the basic harmonic q of the superheating disturbance and the frequency shift $\omega-\omega^0$ as functions of St_m and the supercriticality for the bubble growth rate described by equation (12); solid curve, $St = St_m$; dashed curve, $St = 0.1$.

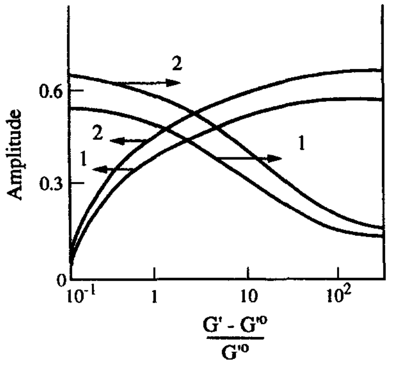

Fig. 4. The amplitude and frequency of self-oscillations of superheating u/u_s as functions of the supercriticality for $St = 0.5$, $St_m = 1.0$; 1, equation (12); 2, equation (11).

tallization from supersaturated solutions and supercooled melts, the region of the existence of which is much more wide; see ref. [30]). Thus, the slightly nonlinear boiling regime with a small relative amplitude can hardly be realized in practice. However, the results of investigation of slightly nonlinear self-oscillations prove to be useful for the numerical analysis of the evolution equation (19) when they are used as the test ones.

The results of numerical analysis of the amplitude and frequency of self-oscillations of relative superheating and the mean (averaged with respect to time) superheating are shown in Figs. 4 and 5. One can easily establish a correlation between the amplitude and the frequency of oscillations, which is an essential feature of nonlinear oscillations. It is worth nothing that the mean superheating decreases with a growth of the supercriticality, which is in full conformity with the result of the linear analysis.

Important technological characteristics of self-oscillating boiling regimes are the oscillations of the concentration of bubbles and the total vapour mass in the power equipment. Since the bubble size distribution is known [equation (15)], calculation of these characteristics is reduced to the calculation of the corresponding moments of the bubble size distribution function.

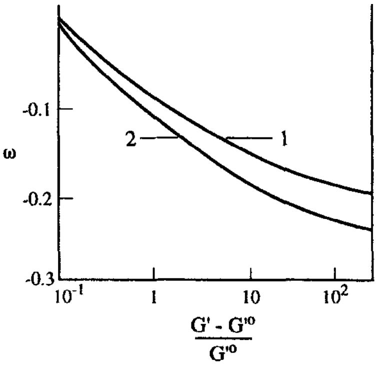

Fig. 5. The amplitude and frequency of self-oscillations of the mean superheating $\langle u/u_s \rangle - 1$ as functions of the supercriticality for $St = 0.5$, $St_m = 1.0$; 1, equation (12); 2, equation (11).

The above analysis evidence that the periodic oscillations of important characteristics of bulk boiling represent the rule rather than the exception. The results obtained give one an opportunity to calculate theoretically the amplitude, the frequency and the form of oscillating cycles depending on physical and regime parameters.

Unfortunately, experimental verification of the results obtained is complicated now by the absense of experiments studying just the bulk boiling in a sufficiently large volume at a full exclusion of heterogeneous boiling; in any case, the authors do not know of such experiments. However, the practical importance of the problem reflected above makes the arrangement of such experiments highly desirable.

PERIODIC REGIMES OF NUCLEATE, FILM AND TRANSITION BOILING ON A SUPERHEATED SURFACE

The model

Consider a semi-infinite volume of liquid being in contact with a heated plate of finite thickness. In those cases when the surface temperature is higher than the saturation temperature, the maximum drop of the temperature occurs in a thin boundary layer. Metastability in the boundary layer activates nuclei existing in the cavities on the surface. The number of virtual nucleation centres, i.e. stable nuclei, at high heat fluxes is a strongly nonlinear function of the superheating and increases rapidly with a growth of the superheating. This causes a spontaneous generation of bubbles at sufficiently high superheating. At the initial stage the bubbles are small, and the heat released cannot essentially reduce the superheating. After a while, the rate of heat release increases. Thus, the boundary layer cools and the surface temperature drops. Having reached the departure size, the bubbles quickly leave the heating surface, thus allowing for the inflow of relatively cold masses of liquid from the volume, which results in the cessation of nucleation. The condition determining the possibility of such a contact is retaining of thermodynamic stability of the boundary layer. (If the surface temperature is higher than the temperature of spinodal, the spontaneous boiling of a thin liquid layer and formation of a vapour film takes place; this is the so-called thermodynamic boiling crisis [32].)

After some period of time the boundary layer is heated again to the temperature corresponding to intensive activation of cavities, and the cycle is repeated. Hence, an original regime of pulsating nucleate boiling, which was observed in experiments repeatedly [33], is established. At a sufficiently dense distribution of vapour generation centres or large departure diameter the bubbles may combine before reaching the departure diameter. Thus, the film boiling may form, since the heat from the plate is transfered directly to the vapour film, separating the liquid and the heating surface, and favours the evaporation of liquid from the vapour–liquid interface. As the hydrodynamic instability of the film grows, the amplitude of the interface surface oscillations becomes comparable with the dominant wave amplitude, i.e. a wave of low frequency for which the image point part of the frequency is maximum. Thus, the necessary condition for liquid separated from the heating surface by the vapour film, to contact with the surface is hydrodynamic instability of the film and thermodynamic stability of liquid at the place of contact. It is necessary to determine that condition which is fulfilled under smaller temperature gradient; for boiling on the horizontal plate the possibility of contact is limited by the last condition.

The boiling process described above is the transient boiling in the unstable regime, when the heating surface contacts alternatively with vapour and liquid, i.e. the so-called dry patch boiling with repeated rewetting.

The analysis of nucleate, film and transient boiling was carried out on the basis of the following model. To describe the heat transfer in the plate, use was made of the heat conduction equation for the temperature $T_1(z,t)$ in the longitudinal section at the distance z from the lower surface of the plate

$$\rho_0 c_1 \frac{\partial T_1}{\partial t} = \lambda_0 \frac{\partial^2 T_1}{\partial z^2} \quad 0 \leqslant z \leqslant \delta. \tag{24}$$

Equation (24) is supplemented with boundary conditions, the first describing the heat flux from an external source and the second corresponding to the equality of the wall temperature and that of the liquid on the heating surface

$$\lambda_0 \left. \frac{\partial T_1}{\partial z} \right|_{z=0} = -q$$

$$T_1(\delta, t) = T_2(\delta, t). \tag{25}$$

The initial temperature distribution in the plate can be arbitrary. It is assumed that $T_1(z,0) = T_{10} = \text{const.}$

If the surface temperature is lower than the temperature of activation, one has the condition of balance (conjugation) of heat fluxes

$$\lambda_0 \left. \frac{\partial T_1}{\partial z} \right|_{z=\delta} = \lambda' \left. \frac{\partial T_2}{\partial z} \right|_{z=\delta} \tag{26}$$

(thus, the temperature regime on the heating surface is not set *a priori*, but is determined by the solution of heat conduction equations in the contacting regions).

When there are no bubbles on the heating surface, one uses for the boundary layer the heat conduction equation analogous to equation (24) with the initial condition $T_2(z,0) = T_{20} = \text{const.}$

Let us assume that the distribution density of virtual boiling centres on the surface is high enough to neglect the temperature variations in the horizontal direction. This assumption corresponds actually to negligibly small changes of the liquid temperature in the horizontal direction as compared with those in the vertical one (since the boundary layer, where the temperature gradients are maximum, is very thin). Besides that, the distribution of vapour generation centres is assumed to be homogeneous along the surface. Thus, it is assumed that the heat fluxes released in the boiling process by growing bubbles is uniform along the heating surface.

To calculate the bubble size distribution function, it is possible to use the correlations for the distribution density of active vapour generation centres obtained experimentally [34]

$$n = C\left(\frac{L\rho''\Delta T}{\sigma T_0}\right)^k, \quad C, k = \text{const.} \tag{27}$$

Formula (27) is obtained under the assumption of the activation nature of boiling centres, when their number is a strongly nonlinear function of superheating.

Convective heat transfer in the boundary layer caused by intensive turbulization of liquid by the bubbles is modelled by the heat conduction equation with the effective coefficient

$$\lambda'^* = \lambda' f^*(Re, Pr) \tag{28}$$

being defined by thermohydrodynamic situation in the boundary layer and the thermophysical properties of the liquid.

Taking account of the heat release during the bubble growth modifies the conjugation condition (26), which assumes the form

$$\lambda_0 \frac{\partial T_1}{\partial z} = \lambda'^* \frac{\partial T_2}{\partial z} - f(t,r)\frac{\mathrm{d}Q}{\mathrm{d}t} \quad z = \delta \tag{29}$$

where $\mathrm{d}Q \approx 4\pi\rho'' L r^2\, \mathrm{d}r$ is the heat conducted to the bubble during its growth.

There are many correlations describing the bubble growth rate. If the heat is transferred to the bubble from the liquid in the form of evaporation heat close to the base of the bubble, then [35]

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \hat{\alpha}\frac{\hat{\lambda}'}{\rho'' L}\Delta T \quad \hat{\alpha} \sim 5 \div 10, \quad Ja \ll 1 \tag{30}$$

(the heat is conducted to evaporation zone from the heating surface through the liquid microlayer close to the bubble).

Taking account of the heat transferred to the bubble through the latter interphase surface gives the following expression for the bubble growth [36]

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{a_2}{2r}(\beta_1 Ya + \sqrt{(\beta_1^2 Ja^2 + 2\beta_2 Ja)}). \tag{31}$$

The paper [36] shows that $\beta_1 = 0.3$ and $\beta_2 = 6$ describe experimental data sufficiently well.

It is convenient to introduce new variables and parameters

$$Fo = a_1 t/\delta^2$$

$$\theta_i = (T_i - T_0)/T_0$$

$$\xi = z/\delta$$

$$Ki = q\delta/\lambda_0 T_0$$

$$K = L/c_3 T_0. \tag{32}$$

Then the system modelling the problem assumes the form

$$\frac{\partial\theta_1}{\partial Fo} = \frac{\partial^2\theta_1}{\partial\xi^2}$$

$$\left.\frac{\partial\theta_1}{\partial\xi}\right|_{=0} = -Ki$$

$$\theta_1|_{Fo=0} = \theta_{10}$$

$$\frac{\partial\theta_2}{\partial Fo} = \frac{\partial^2\theta_2}{\partial\xi^2}\frac{a_2}{a_1}$$

$$\theta_1(1, Fo) = \theta_2(1, Fo)$$

$$\left.\frac{\lambda_0\,\partial\theta_1}{\lambda'\,\partial\xi}\right|_{\xi=1} = \left.\frac{\partial\theta_2}{\partial\xi}\right|_{\xi=\perp} \tag{33}$$

(bubbles are absent),

$$\frac{\partial\theta_1}{\partial\xi} = \frac{\lambda'^*\,\partial\theta_2}{\lambda_0\,\partial\xi} - \frac{4\pi K\rho'' c_3}{\delta\rho_0 c_1} f(Fo, r) r^2 \frac{\mathrm{d}r}{\mathrm{d}Fo}$$

$$\xi = 1 \tag{34}$$

(heat realease for the bubble growth is taken into account).

According to the model described above, at the moment of bubble departure from the heating surface Fo_* its temperature instantly drops to a certain value θ_2^*. For its approximate calculation it is possible to assume that the total volume of liquid, coming to the heating surface instead of bubbles, is equal to the total volume of removed bubbles V_*. Taking into consideration that the boundary layer is intensively turbulized by moving bubbles (thus the temperature throughout the surface can be regarded as uniform), one obtains

$$\theta_2^* \approx \frac{1}{SD_*}\left[V_*\theta_\infty + (SD_* - V_*) \times \frac{\delta}{D_*}\int_0^{D_*/\delta} \theta_2(\xi, Fo_*)\,\mathrm{d}\xi\right]. \tag{35}$$

The correctness of such a definition increases at a decrease of the dispersivity of the bubble size distribution function.

If bubbles form a continuous vapour film before reaching the departure diameter, it is necessary to take into account that the heat from the plate is transferred to the vapour film, and then is spent for the liquid evaporation from the interphase surface. The initial film thickness is defined by the total volume of bubbles. To describe the film growth, the Stephan condition can be used:

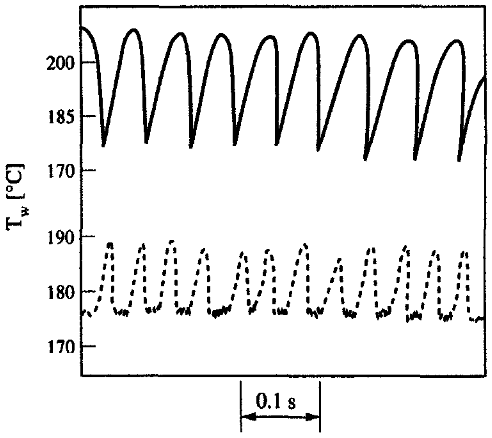

Fig. 6. Self-oscillations in nucleate boiling; solid curve, theory; dashed curve, experiment [37].

$$\lambda'^*\frac{\partial T_2}{\partial z} - \lambda''\frac{\partial T_3}{\partial z} = L\rho''\frac{\mathrm{d}y}{\mathrm{d}t} \quad z = y(t). \tag{36}$$

The condition of contact of hydrodynamically unstable liquid–vapour interface and the heating surface is the equality of the amplitude of the dominant wave [3]

$$A = \frac{0.25\tilde{z}(\rho' g)^2 L}{(-\partial q_w/\partial z)}$$

$$q_w = \lambda_0\frac{\partial T_1}{\partial z} \tag{37}$$

to the thickness of vapour film.

Results and discussion

The problem stated was analysed numerically by using the three-layer finite-difference Dufort–Frankel scheme. The calculation was carried out on a non-uniform grid with the grid fineness enhanced on the boundary surfaces. The results obtained show that in the both cases, corresponding to the pulsating nucleate and unstable transient boiling, the surface temperature oscillations have a typical relaxational character. The relaxational period is small and consists of a few oscillating cycles. The comparison of theoretical results with experiments [37, 38] is illustrated in Figs. 6 and 7. Reference [37] studied self-oscillations of the surface temperature of a copper plate of thickness 0.2 mm for the nucleate boiling of distilled water at $q = 4.57 \times 10^5$ W m^{-2}, which corresponds to $Ki = 2.36 \times 10^{-2}$, $K = 32.75$. Reference [38] analysed the dry patch formed during boiling with repeated rewetting of sodium on a nickel surface at $\delta = 6$ mm, $q = 5.6 \times 10^5$ W m^{-2}, which corresponds to $Ki = 8.42 \times 10^{-2}$, $K = 24.14$. Figures 6 and 7 demonstrate an essential difference between the form of oscillations in nucleate and transition boiling. Self-oscillations in transition boiling have a serrated form,

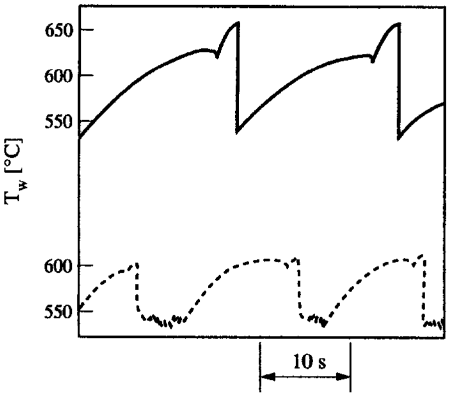

Fig. 7. Self-oscillations at dry patch formed boiling with repeated rewetting; solid curve, theory; dashed curve, experiment [38].

which is connected with the release of latent heat of phase transition in the bubble growth before combining in the film, and then a rapid surface temperature enhancement due to the contact with the vapour film. The oscillation amplitude in transition boiling exceeds that in nucleate boiling.

Determination of the conditions of transfer from nucleate to transition boiling, which corresponds to film formation before bubble departure, is of great interest. The model suggested allows for determining the critical superheating providing the film formation, as a function of coefficient k (see Fig. 8), i.e. the degree of nonlinear dependence of vapour generation centres density on superheating.

CONCLUDING REMARKS

The theory developed gives one an opportunity to predict conditions of transition from stable boiling regimes to unstable ones when the temperature and other characteristics of two-phase systems oscillate. As regards the bulk boiling, the boundaries separating the regions of stability and instability of stationary boiling regimes are obtained in analytical form for various regime and physical parameters. The analysis revealed that stationary regimes of bulk boiling are stable in a very narrow interval of practically possible parameters and, consequently, can be hardly observed in experiments. The instability is caused by the strongly nonlinear dependence of nucleation rate on superheating. As distinct from continuous crystalization studied in the previous paper [30], the instability in the bulk boiling grows rapidly (but it is not the 'hard' type of instability in mathematical sense). The amplitude and frequency of self-oscillations establishing as a result of the instability are calculated for the most typical laws of bubble growth, the peculiarities of which are revealed.

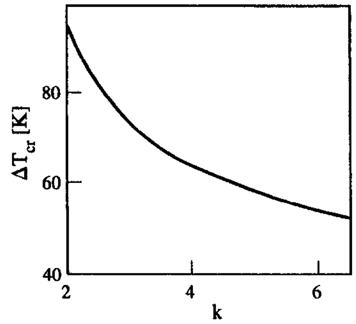

Fig. 8. The critical superheating ΔT_{cr}, separating transition and nucleate boiling for $Ki = 0.05$, $K = 20$.

The model suggested for nucleate and film boiling describes the phenomena of pulsating regime of nucleate boiling and dry patch formed during boiling and burnout, when the dry patch formation and the rewetting are alternatively repeated in intermittent boiling. Determination of conditions of transition to the latter type of film boiling seems to be the most valuable result of the second part of the paper.

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